Measurement of Dispersion Effects on Narrow RF Pulses

In an earlier issue of these TRANSACTIONS, Elliott [1] derived a formula for the degradation of RF pulses due to dispersion in waveguide. This communication describes an experiment conducted to verify his results. An RF pulse generator capable of producing one nanosecond pulse widths was developed to implement the experiment.

The distortion of a modulation envelope in a dispersive transmission line is dependent on the line length, the spectrum width, and the proximity of the carrier frequency to the cutoff frequency. These factors are embodied in Elliott's formula for the output pulse shape produced by transmitting a rectangular RF pulse of width, T, through a section of waveguide of length, L. This formula is:

$$F(t) = \frac{1}{2}\sqrt{\left\{\operatorname{erf}\left[\frac{x+1}{a}\right] - \operatorname{erf}\left[\frac{x-1}{a}\right]\right\}^2 + \left\{C\left[\left(\frac{x+1}{a}\right)^2\right] - S\left[\left(\frac{x+1}{a}\right)^2\right] - C\left[\left(\frac{x-1}{a}\right)^2\right] + S\left[\left(\frac{x-1}{a}\right)^2\right]\right\}^2}$$

where

$$x = \frac{2t}{T}$$

$$a = \frac{4\omega_c}{Tv^2\beta}\sqrt{\frac{L}{2\beta_0}}$$

β_0 = propagation constant at the carrier frequency

$\omega_c = 2\pi f_c$ (where f_c is the guide cutoff frequency)

$v = 1/\sqrt{\mu\epsilon} = 3\times10^8$ meters/sec for air filled guide

erf (p) is the error function $\frac{2}{\sqrt{\pi}}\int_0^p e^{-u^2}du$

$C(p)$ is the Fresnel integral $\frac{\sqrt{2}}{\sqrt{\pi}}\int_0^{\sqrt{p}} \cos u^2 du$

$S(p)$ is the Fresnel integral $\frac{\sqrt{2}}{\sqrt{\pi}}\int_0^{\sqrt{p}} \sin u^2 du$

The parameter a, which appears in Elliott's formula serves as a figure of merit. Decreasing T, increasing the line length, or decreasing the difference between carrier and cutoff frequency will increase a. Calculated waveforms for $a=0.032$, 0.10, 0.32, and 1.0 are given by Elliott. From his figures it can be seen that significant distortion appears for a greater than 0.032. Figure 1 shows the calculated degradation of a rectangular 0.7 ns, 10 Gc/s pulse after passing through two meters of RG52/U waveguide. For this case $a=1.32$. Observe that the pulse width has been significantly increased. This smearing could cause severe intersymbol interference of modulating pulses whenever such pulses are separated by less than $2T$ or could cause a significant loss of time resolution in a radar-type system.

Figure 2 is a block diagram of the test setup used to verify Elliott's results. The amplitude modulator consists of a pulse generator, high-speed diode switch, and circulator. Various lengths of waveguide and coaxial transmission line were inserted between the circulator and oscillator and the resulting wave shapes were observed on a sampling osiclloscope. The oscillator was used to prevent pulse distortion due to multiple reflections.

The pulse generator was developed at Bendix Research Labs. and produces 1 ns pulses with a peak amplitude of greater than 2 volts at 100 mA. The generator consists of snap-off diodes driven by a sine wave clock. The output capability of the pulse generator is sufficient to drive a Philco P-901 switch with a L-4120 diode. The pulse shape at the diode is shown in Fig. 3. Since the pulses were extremely narrow, the pulse generator was constructed as an integral part of the microwave diode holder. This eliminated excessive ringing caused by the inductance of the BNC connector originally mounted on the switch.

The pulse demodulator was a broadband waveguide crystal detector employing an MA408B diode. The bypass capacitance was reduced to 1 to 2 picofarads to lower the detection time constant. An oscilloscope cable was connected directly to the diode contact pin to eliminate distortion produced by the BNC connector.

Figure 4(a) shows the detected waveform with only the isolator between the detector and diode switch. The main cause of the difference between the modulating and detected pulses is the RF switch. The diode in this switch has a nonzero reaction time and has different impedance characteristics during turn-on and turn-off. It was established that the detector was operating near its square law range and that the detected pulse shape is close to the square of the modulation envelope. Figure 4 can be taken as a reference and the degradation in the detected pulse due to dispersion can be observed.

Figure 4(b) shows the output pulse when one meter of RG52/U waveguide is inserted between modulator and isolator. Some stretching has occurred and the backswing has increased. These effects are predictable from the calculated pulse shape shown in Fig. 1. Quantitative comparisons are difficult because the experimental pulse shape is not rectangular. However, good agreement is found between values of a and measured peak amplitudes if T is assumed to be 0.7 ns. This is approximately the width of the RF envelope at the half-voltage points.

For $T=0.7$ ns and $L=1$ meter, $a=0.93$. The theoretical peak output voltage for $a=0.93$ and square law detection would be $0.758E_0$, where E_0 is the pulse height for $a=0$. This agrees very well with the ratio between pulse heights in Fig. 4(a) and (b) which is approximately 0.7.

Figure 4(c) shows the output pulse shape when two meters of RG52/U are inserted between the isolator and modulator. Considerable pulse stretching has occurred and the backswing is as large as the main pulse. Using $T=0.7$ ns, a is 1.32. The theoretical amplitude corresponding to $a=1.32$ is $0.5E_0$. Again there is good agreement between theory and experiment.

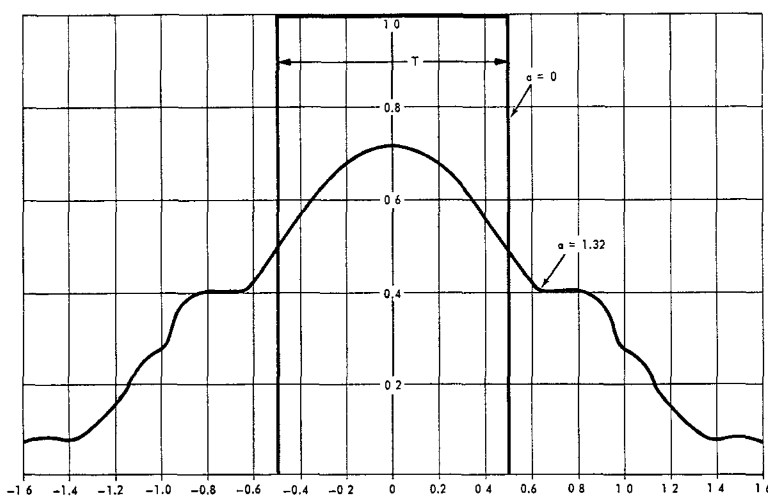

Fig. 1. Theoretical pulse degradation.

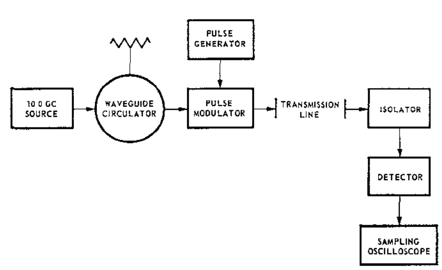

Fig. 2. Block diagram of test setup.

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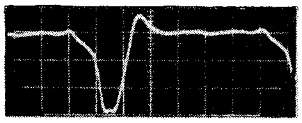

Fig. 3. Pulse generator output (modulating signal).

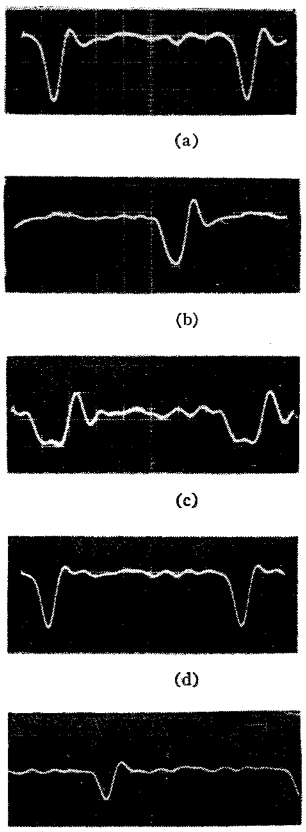

Fig. 4. Detector outputs. (a) Detected pulses for $L=0$. (b) $L=1$ meter of RG52/U. (c) $L=2$ meters of RG52/U. (d) Reference $L=0$ meters of RG9A/U. (e) $L=2$ meters of RG9A.

Figure 4(d) is the output pulse observed when coaxial to waveguide adaptors were inserted between isolator and modulator. This pulse is the reference for the coaxial line. Figure 4(e) is the output pulse observed when two meters of RG9A/U were inserted between the adaptors. No change in pulse shape should occur because TEM or coaxial transmission is not dispersive. The output pulse shows that the only effect is a 3 dB loss in power. Since RG9A/U cable has an attenuation of 0.5 dB/ft at 10 Gc/s, the detected output amplitude agrees very well with theory.

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[1] Elliott, R. S., Pulse waveform degradation due to dispersion in waveguide, *IRE Trans. on Microwave Theory and Techniques*, vol MTT-5, Oct 1957, pp 254–257.
[2] NBS, Tables of the error function and its derivative, *Appl. Math. Ser.*, vol 41, 1954.
[3] Pearcey, T., *Table of the Fresnel Integral to Six Decimal Places*. London: Cambridge, 1956.

Maximally Flat Bandwidth of a Nondegenerate Parametric Amplifier with Double Tuned Signal Circuit and Single Tuned Idler Circuit

The advantages of an amplifier employing a single tuned idler circuit have been discussed by DeJager.[1] Using a single tuned idler circuit, DeJager has calculated the maximum bandwidth obtainable with a single tuned signal circuit and also the maximum limiting flat bandwidth.

It is the purpose of this letter to present the results of the extension of DeJager's work to the case of a single tuned idler circuit and double tuned signal circuit. As shown by DeJager, the input impedance of the single tuned amplifier is given by

$$\overline{Z_A} = R_A\left(2j\frac{\Delta\omega}{\omega_1}Q_1 - \frac{1}{1+2j\dfrac{\Delta\omega}{\omega_1}Q_2}\right) \quad (1)$$

If a shunt tuned circuit resonant at the signal frequency is connected across the input terminals (see Fig. 1), the input admittance is given by

$$Y_{\text{in}} = Y + \frac{1}{\overline{Z_A}} \quad (2)$$

where

$$Y \cong j\frac{2Q}{R_A}\frac{\Delta\omega}{\omega_1}$$

and

$$Q = \omega_1 R_A C$$

Therefore

$$Y_{\text{in}} = -\frac{1}{R_A}\left[\frac{1-QQ_1v^2-j(Q_1Q_2Qv^3+Qv-Q_2v)}{1+Q_1Q_2v^2-jQ_1v}\right]$$

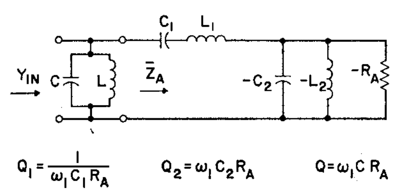

Fig. 1. Equivalent signal circuit near resonance.

where

$$v = 2\frac{\Delta\omega}{\omega_1}$$

The power gain of the amplifier is given by

$$g^2(v) = \frac{[Y_0R_A + 1 - (QQ_1 - Y_0R_AQ_1Q_2)v^2]^2 + v^2[Y_0R_AQ_1 - Q_2 + Q + Q_1Q_2Qv^2]^2}{[Y_0R_A - 1 + (QQ_1 + Y_0R_AQ_1Q_2)v^2]^2 + v^2[Y_0R_AQ_1 + Q_2 - Q - Q_1Q_2Qv_o^2]^2}$$

Let

$$g^2(o) = \left(\frac{Y_0R_A+1}{Y_0R_A-1}\right)^2 = g^2$$

and define $Q_M{}^2 = Q_1Q_2$, $q^2 = Q_2/Q_1$ after DeJager and also let

$$X = \frac{Q}{Q_M}\frac{1}{q}, \qquad \alpha = Q_M v$$

Then

$$g^2(\alpha) = \frac{A + B\alpha^2 + C\alpha^4 + D\alpha^6}{E + B\alpha^2 + C\alpha^4 + D\alpha^6}$$

where

$$A = \left(\frac{2g}{g+1}\right)^2$$

$$B = \frac{1}{q^2}\left\{\left(\frac{g-1}{g+1}\right)^2 + 2\left[\left(\frac{g-1}{g+1}\right)^2 - X\right]q^2 + (1-X)^2q^4\right\}$$

$$C = X^2 + \left(\frac{g-1}{g+1}\right)^2 - 2X(1-X)q^2$$

$$D = X^2q^2$$

$$E = \left(\frac{2}{g+1}\right)^2$$

For maximally flat gain, we must set as many derivatives of $g^2(\alpha)$, evaluated at $\alpha=0$, to zero as possible. Setting

$$\frac{d^2[g^2(0)]}{d\alpha^2} = \frac{d^4[g^2(0)]}{d\alpha^4} = 0,$$

we must have

$$B = C = 0$$

Thus

$$q^4(1-X)^2 - 2\left[X - \left(\frac{g-1}{g+1}\right)^2\right]q^2 + \left(\frac{g-1}{g+1}\right)^2 = 0$$

$$X^2 + \left(\frac{g-1}{g+1}\right)^2 - 2X(1-X)q^2 = 0$$

The above two equations can be satisfied simultaneously for

$$X = \frac{g-1}{g+1}, \qquad q^2 = \frac{g-1}{2}$$

Therefore, we have

$$q_{\text{opt}} = \sqrt{\frac{g-1}{2}}$$

$$\frac{Q}{QM} = \left(\frac{g-1}{g+1}\right)\sqrt{\frac{g-1}{2}}$$

$$g^2(\alpha)\,|_{\text{opt}} = \frac{g^2 + \left(\dfrac{g-1}{2}\right)^3\alpha^6}{1 + \left(\dfrac{g-1}{2}\right)^3\alpha^6}$$

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[1] DeJager, J. T., Maximum bandwidth performance of a nondegenerate parametric amplifier with single-tuned idler circuit, *IEEE Trans. on Microwave Theory and Techniques*, vol MTT-12, Jul 1964, pp 459–467.